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On the Determination and Solution of the Metacyclic Quintic Equations with Rational Coefficients.

BY J. C. GLASHAN.

(The following paper is in tardy fulfillment of a promise made to the late Professor George Paxton Young. See *American Journal of Mathematics*, vol. VI, page 114).

§1. If $c\ d\ e\ f$ are given rational numbers, and if

$$
\begin{aligned}
A &\equiv 3c^2 + e,\\
B &\equiv 15c^4 - 2c^2e + 8cd^2 - 2df + 3e^2,\\
C &\equiv 25c^6 - 35c^4e + 40c^3d^2 + 2c^2df + 11c^2e^2 - 28cd^2e - cf^2 + 16d^4 + 2def - e^3,\\
D &\equiv 3456c^5f^2 - 11520c^4def + 6400c^4e^3 + 5120c^3d^3f - 3200c^3d^2e^2 - 1440c^3ef^2\\
&\quad + 2640c^2d^2f^2 + 4480c^2de^2f - 2560c^2e^4 - 10080cd^3ef + 5760cd^2e^3 - 120cdf^3\\
&\quad + 160ce^2f^2 + 3456d^5f - 2160d^4e^2 + 360d^2ef^2 - 640de^3f + 256e^5 + f^4,
\end{aligned}
$$

the quintic equation

$$x^5 + 10cx^3 + 10dx^2 + 5ex + f = 0 \tag{1}$$

will be solvable by radicals if the sextic equation

$$(u^3 - 5Au^2 + 5Bu - 5C)^2 - Du = 0 \tag{2}$$

have a rational root, but if the sextic (2) have not a rational root, the quintic (1) will not be solvable by radicals. (In examining equation (2) for a rational root, only common factors of C and D of the form $m^2 + n^2$ or the form m^2, need be tried as values of u.)

If the sextic (2) have a rational root, denote it by u_1, and let

$$
\begin{aligned}
&E \equiv 2cd + f,\\
&F \equiv 10c^3d + 11c^2f - 18cde + 12d^3 + ef,\\
&G \equiv 50c^5d - 59c^4f + 20c^3de + 40c^2d^3 + 42c^2ef - 48cd^2f - 38cde^2 + 44d^3e - df^2 + e^2f,\\
&\tau_2 = \tfrac{1}{5}\sqrt{u_1}.\\
&\tau_3 = \frac{(Eu_1^2 - 2Fu_1 + G)\,\tau_2}{u_1^3 - 3Au_1^2 + Bu_1 + C},\\
&\alpha_1 = \tfrac{1}{2}\left[-(d - \tau_3) + \sqrt{\{(d - \tau_3)^2 + 4(c - \tau_2)(c^2 - \tau_2^2)\}}\right],\\
&\alpha_2 = \tfrac{1}{2}\left[-(d + \tau_3) - \sqrt{\{(d + \tau_3)^2 + 4(c + \tau_2)(c^2 - \tau_2^2)\}}\right],\\
&\alpha_3 = \tfrac{1}{2}\left[-(d + \tau_3) + \sqrt{\{(d + \tau_3)^2 + 4(c + \tau_2)(c^2 - \tau_2^2)\}}\right],\\
&\alpha_4 = \tfrac{1}{2}\left[-(d - \tau_3) - \sqrt{\{(d - \tau_3)^2 + 4(c - \tau_2)(c^2 - \tau_2^2)\}}\right].
\end{aligned}
$$

and $\omega^5 - 1 = 0,$

then will

$$x = \omega\left\{\frac{\alpha_1^2\alpha_3}{(c + \tau_2)^2}\right\}^{\frac{1}{5}} + \omega^2\left\{\frac{\alpha_3^2\alpha_4}{(c - \tau_2)^2}\right\}^{\frac{1}{5}} + \omega^3\left\{\frac{\alpha_2^2\alpha_1}{(c - \tau_2)^2}\right\}^{\frac{1}{5}} + \omega^4\left\{\frac{\alpha_4^2\alpha_2}{(c + \tau_2)^2}\right\}^{\frac{1}{5}}.$$

§2. The preceding propositions may be proved as follows:

Let $x = \omega y_1 + \omega^2 y_2 + \omega^3 y_3 + \omega^4 y_4,$

in which $\omega^5 - 1 = 0,$

then will

$$x^5 - 10\sigma_2 x^3 - 10\sigma_3 x^2 - 5(2\sigma_4 - \sigma_2^2 - 3\tau_2^2)\,x - 2\sigma_5 + 20\tau_2\tau_3 = 0, \tag{3}$$

in which

$$
\begin{aligned}
2\sigma_2 &= y_1y_4 + y_2y_3, & 2\tau_2 &= y_1y_4 - y_2y_3,\\
2\sigma_3 &= y_1^2y_3 + y_2^2y_1 + y_3^2y_4 + y_4^2y_2, & 2\tau_3 &= y_1^2y_3 - y_2^2y_1 - y_3^2y_4 + y_4^2y_2,\\
2\sigma_4 &= y_1^3y_2 + y_2^3y_4 + y_3^3y_1 + y_4^3y_3,\\
2\sigma_5 &= y_1^5 + y_2^5 + y_3^5 + y_4^5.
\end{aligned}
$$

Comparing equations (1) and (3), gives us at once

$$
\begin{aligned}
\sigma_2 &= -c,\\
\sigma_3 &= -d,\\
2\sigma_4 - \sigma_2^2 - 3\tau_2^2 &= -e,\\
2\sigma_5 - 20\tau_2\tau_3 &= -f.
\end{aligned}
$$

To these we add

$$\begin{aligned}
\sigma_3^2 - \tau_3^2 &= 2(\sigma_2\sigma_4 - \tau_2\tau_4),\\
\sigma_4^2 - \tau_4^2 &= 2\sigma_2(\sigma_3^2 + \tau_3^2) - 4\tau_2\sigma_3\tau_3 - 4(\sigma_2^2 - \tau_2^2)^2,\\
(\sigma_3 + \tau_3)(\sigma_4 + \tau_4) &= (\sigma_2 - \tau_2)(\sigma_5 + \tau_5) + (\sigma_2 + \tau_2)^2(\sigma_3 - \tau_3),\\
(\sigma_3 - \tau_3)(\sigma_4 - \tau_4) &= (\sigma_2 + \tau_2)(\sigma_5 - \tau_5) + (\sigma_2 - \tau_2)^2(\sigma_3 + \tau_3),
\end{aligned}$$

in which

$$\begin{aligned}
2\tau_4 &= y_1^3 y_2 - y_2^3 y_4 - y_3^3 y_1 + y_4^3 y_3,\\
2\tau_5 &= y_1^5 \quad - y_2^5 \quad - y_3^5 \quad + y_4^5.
\end{aligned}$$

On eliminating σ_2, σ_3, σ_4, σ_5, τ_3, τ_4 and τ_5 from these equations and writing u for $25\tau_2^2$, we obtain

$$\begin{aligned}
[\{&u^3 - 3(3c^2 + e)u^2 + (15c^4 - 2c^2e + 8cd^2 - 2df + 3e^2)u\\
&+ 25c^6 - 35c^4e + 40c^3d^2 + 2c^2df + 11c^2e^2 - 28cd^2e - cf^2\\
&+ 16d^4 + 2def - e^3\} \times\\
\{&cu^4 - (20c^3 + 8ce - d^2)u^3 + (150c^5 + 20c^3e + 55c^2d^2 - 14cdf\\
&+ 38ce^2 - 7d^2e + f^2)u^2 - (500c^7 - 400c^5e + 625c^4d^2 + 36c^3df\\
&+ 100c^3e^2 - 270c^2d^2e + 24c^2f^2 + 200cd^4 - 104cdef + 56ce^3\\
&+ 66d^3f - 31d^2e^2 + ef^2)u + 25(c^3 - ce + d^2)(25c^6 - 35c^4e + 40c^3d^2\\
&+ 2c^2df + 11c^2e^2 - 28cd^2e - cf^2 + 16d^4 + 2def - e^3)\}\\
&- \{(2cd + f)u^2 - 2(10c^3d + 11c^2f - 18cde + 12d^3 + ef)u + 50c^5d - 59c^4f\\
&+ 20c^3de + 40c^2d^3 + 42c^2ef - 48cd^2f - 38cde^2 + 44d^3e - df^2 + e^2f\}^2 u]\\
&\div (cu + c^3 - ce + d^2) = 0.
\end{aligned}$$

On performing the indicated multiplications and division, the resulting sextic equation may easily be reduced to the form in which it has been given in equation (2) of §1, viz.:

$$(u^3 - 5Au^2 + 5Bu - 5C)^2 - Du = 0.$$

In the course of the elimination, the equation

$$\tau_3 = \frac{(Eu^2 - 2Fu + G)\tau_2}{u^3 - 3Au^2 + Bu + C}$$

is obtained, and thus τ_3 is given in terms of τ_2.

If τ_2 and τ_3 have been determined, we have

$$y_1 y_4 = \sigma_2 + \tau_2 = -(c - \tau_2),$$
$$y_2 y_3 = \sigma_2 - \tau_2 = -(c + \tau_2),$$
$$y_1^2 y_3 + y_4^2 y_2 = \sigma_3 + \tau_3 = -(d - \tau_3),$$
$$y_2^2 y_1 + y_3^2 y_4 = \sigma_3 - \tau_3 = -(d + \tau_3),$$

$$\therefore \quad y_1^2 y_3 - y_4^2 y_2 = \surd\{(d - \tau_3)^2 + 4(c - \tau_2)^2(c + \tau_2)\},$$
$$y_2^2 y_1 - y_3^2 y_4 = \surd\{(d + \tau_3)^2 + 4(c + \tau_2)^2(c - \tau_2)\},$$

$$\therefore \quad y_1^2 y_3 = \tfrac{1}{2}[-(d - \tau_3) + \surd\{(\mathrm{d} - \tau_3)^2 + 4(c - \tau_2)(c^2 - \tau_2^2)\}] = \alpha_1,$$
$$y_4^2 y_2 = \tfrac{1}{2}[-(d - \tau_3) - \surd\{(d - \tau_3)^2 + 4(c - \tau_2)(c^2 - \tau_2^2)\}] = \alpha_4,$$
$$y_2^2 y_1 = \tfrac{1}{2}[-(d + \tau_3) + \surd\{(d + \tau_3)^2 + 4(c + \tau_2)(c^2 - \tau_2^2)\}] = \alpha_3,$$
$$y_3^2 y_4 = \tfrac{1}{2}[-(d + \tau_3) - \surd\{(d + \tau_3)^2 + 4(c + \tau_2)(c^2 - \tau_2^2)\}] = \alpha_2,$$

$$\therefore \quad y_1^5 = \frac{\alpha_1^2 \alpha_3}{(c + \tau_2)^2}, \qquad y_4^5 = \frac{\alpha_4^2 \alpha_2}{(c + \tau_2)^2},$$
$$y_2^5 = \frac{\alpha_3^2 \alpha_4}{(c - \tau_2)^2}, \qquad y_3^5 = \frac{\alpha_2^2 \alpha_1}{(c - \tau_2)^2},$$

and
$$x = \omega y_1 + \omega^2 y_2 + \omega^3 y_3 + \omega^4 y_4.$$

EXAMPLES.

1.
$$x^5 + 3x^2 + 2x - 1 = 0,$$

$$\therefore \quad c = 0, \quad d = .3, \quad e = .4, \quad f = -1,$$

∴ the critical sextic is

$$(u^3 - 2u^2 + 5.4u + .872)^2 - 17.672u = 0,$$

$$\therefore \quad u_1 = .2,$$

$$\therefore \quad \tau_2 = \tfrac{1}{25}\surd 5 \text{ and } \tau_3 = \tfrac{7}{50}\surd 5,$$

$$\therefore \quad \alpha_1 = \frac{1}{500}[-75 + 35\surd 5 + \surd\{470(25 - 11\surd 5)\}],$$
$$\alpha_2 = \frac{1}{500}[-75 - 35\surd 5 - \surd\{470(25 + 11\surd 5)\}],$$
$$\alpha_3 = \frac{1}{500}[-75 - 35\surd 5 + \surd\{470(25 + 11\surd 5)\}],$$
$$\alpha_4 = \frac{1}{500}[-75 + 35\surd 5 - \surd\{470(25 - 11\surd 5)\}],$$

$$\therefore \qquad y_1^5 = \frac{13}{500}(15-7\sqrt{5}) + \sqrt{\left\{\left(\frac{13}{500}\right)^2(15-7\sqrt{5})^2 - \left(\frac{\sqrt{5}}{25}\right)^5\right\}},$$

$$y_2^5 = \frac{13}{500}(15+7\sqrt{5}) - \sqrt{\left\{\left(\frac{13}{500}\right)^2(15+7\sqrt{5})^2 + \left(\frac{\sqrt{5}}{25}\right)^5\right\}},$$

$$y_3^5 = \frac{13}{500}(15+7\sqrt{5}) + \sqrt{\left\{\left(\frac{13}{500}\right)^2(15+7\sqrt{5})^2 + \left(\frac{\sqrt{5}}{25}\right)^5\right\}},$$

$$y_4^5 = \frac{13}{500}(15-7\sqrt{5}) - \sqrt{\left\{\left(\frac{13}{500}\right)^2(15-7\sqrt{5})^2 - \left(\frac{\sqrt{5}}{25}\right)^5\right\}},$$

and $$x = \omega y_1 + \omega^2 y_2 + \omega^3 y_3 + \omega^4 y_4.$$

The value of y_1^5 may also be written

$$y_1^5 = \frac{13}{500}(15-7\sqrt{5}) + \frac{1}{2500}\sqrt{\{94(21125-9439\sqrt{5})\}},$$

with corresponding values of y_2^5, y_3^5, y_4^5.

(This is the resolvent quintic of the modular equation of the 47th order. It was solved by Professor G. P. Young in the *American Journal of Mathematics*, vol. X, pp. 108–110.)

2. $$x^5 - 10x^3 - 20x^2 - 1505x - 7412 = 0,$$

$$\therefore \qquad c = -1, \quad d = -2, \quad e = -301, \quad f = -7412,$$

$\therefore$ the critical sextic is

$$(u^3 + 1490u^2 + 1213700u - 371439000)^2 - 1883801304320000u = 0,$$

$$\therefore \quad u_1 = 50,$$

$$\therefore \quad \tau_2 = -\sqrt{2} \quad \text{and} \quad \tau_3 = 8\sqrt{2},$$

$$\therefore \quad \alpha_1 = 1 + 4\sqrt{2} + \sqrt{(34 + 7\sqrt{2})},$$

$$\alpha_2 = 1 - 4\sqrt{2} - \sqrt{(34 - 7\sqrt{2})},$$

$$\alpha_3 = 1 - 4\sqrt{2} + \sqrt{(34 - 7\sqrt{2})},$$

$$\alpha_4 = 1 + 4\sqrt{2} - \sqrt{(34 + 7\sqrt{2})},$$

$$\therefore \quad y_1^5 = 9(197 - 139\sqrt{2}) + \sqrt{\{81(197 - 139\sqrt{2})^2 - (1 - \sqrt{2})^5\}},$$

$$y_2^5 = 9(197 + 139\sqrt{2}) - \sqrt{\{81(197 + 139\sqrt{2})^2 - (1 + \sqrt{2})^5\}},$$

$$y_3^5 = 9(197 + 139\sqrt{2}) + \sqrt{\{81(197 + 139\sqrt{2})^2 - (1 + \sqrt{2})^5\}},$$

$$y_4^5 = 9(197 - 139\sqrt{2}) - \sqrt{\{81(197 - 139\sqrt{2})^2 - (1 - \sqrt{2})^5\}},$$

and $x = \omega y_1 + \omega^2 y_2 + \omega^3 y_3 + \omega_4 y_4.$

(This equation was solved by Professor G. P. Young in the *American Journal of Mathematics*, vol. X, pp. 115, 116.)

3. $$x^5 - 4x^4 + 4x^3 - 5x^2 + 12x - 1 = 0.$$

Let $$x = \tfrac{1}{5}(z + 4),$$

$$\therefore \quad z^5 - 60z^3 - 705z^2 + 3460z + 19179 = 0,$$

$$\therefore \quad c = -6, \quad d = -70.5, \quad e = 692, \quad f = 19179,$$

$\therefore$ the critical sextic is

$$(u^3 - 4000u^2 + 19359375u - 4982421875)^2 - 550429992675781 25u = 0,$$

$$\therefore \quad u_1 = 125,$$

$$\therefore \quad \tau_2 = \surd 5, \qquad \tau_3 = \tfrac{83}{2}\surd 5,$$

$$\therefore \quad \alpha_1 = \tfrac{1}{4}\{141 + 83\surd 5 + \surd(51350 + 22910\surd 5)\},$$
$$\alpha_2 = \tfrac{1}{4}\{141 - 83\surd 5 - \surd(51350 - 22910\surd 5)\},$$
$$\alpha_3 = \tfrac{1}{4}\{141 - 83\surd 5 + \surd(51350 - 22910\surd 5)\},$$
$$\alpha_4 = \tfrac{1}{4}\{141 + 83\surd 5 - \surd(51350 + 22910\surd 5)\},$$

$$\therefore \quad y_1^5 = -\tfrac{1}{4}(15029 + 7135\surd 5) - \surd\{\tfrac{1}{16}(15029 + 7135\surd 5)^2 - (6 + \surd 5)^5\},$$
$$y_2^5 = -\tfrac{1}{4}(15029 - 7135\surd 5) - \surd\{\tfrac{1}{16}(15029 - 7135\surd 5)^2 - (6 - \surd 5)^5\},$$
$$y_3^5 = -\tfrac{1}{4}(15029 - 7135\surd 5) + \surd\{\tfrac{1}{16}(15029 - 7135\surd 5)^2 - (6 - \surd 5)^5\},$$
$$y_4^5 = -\tfrac{1}{4}(15029 + 7135\surd 5) + \surd\{\tfrac{1}{16}(15029 + 7135\surd 5)^2 - (6 + \surd 5)^5\},$$

and

$$x = \tfrac{1}{5}(4 + \omega y_1 + \omega^2 y_2 + \omega^3 y_3 + \omega^4 y_4).$$

The value of y_1^5 may also be written

$$y_1^5 = -\tfrac{1}{4}(15029 + 7135\surd 5) - \tfrac{15}{4}\surd\{158(13505 + 6029\surd 5)\},$$

with corresponding values of y_2^5, y_3^5, y_4^5.

(This is the resolvent quintic of the modular equation of the 79th order.)

§3. In the preceding sections it has been assumed that the quintic is of the form

$$x^5 + 10cx^3 + 10dx^2 + 5ex + f = 0.$$

If we consider the full-termed form

$$ax^5 + 5bx^4 + 10cx^3 + 10dx^2 + 5ex + f = 0,$$

the course of the solution will follow that in the preceding investigation, step by step, and may be summarized as follows:

Write

$$\begin{aligned}
H_2 &= ac - b^2, \\
H_3 &= a^2d - 3abc + 2b^3, \\
H_4 &= a^2(ae - 4bd + 3c^2), \\
H_5 &= a^2\,(a^2f - 5abe + 2acd + 8b^2d - 6bc^2), \\
H_6 &= H_2H_4 - H_3^2 - 4H_2^3, \\
H_7 &= H_2H_5 - H_3H_4, \\
H_8 &= H_3H_5 + 6H_2H_6 - H_4^2 + 4H_2^2H_4, \\
H_9 &= 2H_2H_7 - 3H_3H_6, \\
H_{10} &= H_5^2 + 12H_4H_6 + 4H_2H_4^2, \\
H_{12} &= H_2H_{10} - 2H_4H_8 + 9H_6^2 - H_4^3, \\
H_{13} &= H_5H_8 + 2H_4H_9 + 24H_2^5H_3, \\
H_{20} &= 2H_9\,(H_4H_7 - 3H_5H_6) + H_8\,(2H_4H_8 - H_5H_7 - 15H_6^2) - 3H_4^2(H_6^2 + 8H_2^6), \\
ax &= -b + \omega y_1 + \omega^2 y_2 + \omega^3 y_3 + \omega^4 y_4, \\
&\qquad \omega^5 - 1 = 0,
\end{aligned}$$

and as before,

$$\begin{aligned}
2\sigma_2 &= y_1y_4 + y_2y_3, \\
2\sigma_3 &= y_1^2y_3 + y_2^2y_1 + y_3^2y_4 + y_4^2y_2, \\
2\sigma_4 &= y_1^3y_2 + y_2^3y_4 + y_3^3y_1 + y_4^3y_3, \\
2\sigma_5 &= y_1^5 + y_2^5 + y_3^5 + y_4^5, \\
2\tau_2 &= y_1y_4 - y_2y_3, \\
2\tau_3 &= y_1^2y_3 - y_2^2y_1 - y_3^2y_4 + y_4^2y_2, \\
u &= 25\tau_2^2,
\end{aligned}$$

then will

$$\begin{aligned}
\sigma_2 + H_2 &= 0, \\
\sigma_3 + H_3 &= 0, \\
2\sigma_4 - \sigma_2^2 - 3\tau_2^2 + H_4 - 3H_2^2 &= 0, \\
2\sigma_5 - 20\tau_2\tau_3 + H_5 - 2H_2H_3 &= 0.
\end{aligned}$$

Eliminating σ_2, σ_3, σ_4 and σ_5, we obtain

$$\tau_3 = \frac{\{H_5u^2 - 2\,(4H_9 + H_4H_5)\,u - H_{13} + 50H_6H_7\}\,\tau_2}{u_3 - 3H_4u^2 - (2H_8 - H_4^2)\,u - H_{12} + 25H_6^2}$$

and

$$\begin{aligned}
&[\{u^3 - 3H_4u^2 - (2H_8 - H_4^2)\,u - H_{12} + 25H_6^2\} \\
&\quad \times \{H_2u^4 - (H_6 + 7H_2H_4)\,u^3 + (H_{10} - 18H_2H_8 - 5H_4H_6 + 9H_2H_4^2)\,u^2 \\
&\qquad - (H_4H_{10} + 21H_7^2 - 66H_6H_8 - 26H_4^2H_6 + 464H_2H_6^2)\,u + 25H_6\,(H_{12} - 25H_6^2)\} \\
&\quad - \{H_5u^2 - 2\,(4H_9 + H_4H_5)\,u - H_{13} + 50H_6H_7\}^2\,u] \div (H_2u - H_6) = 0,
\end{aligned}$$

which can be reduced to the form

$$\{u^3 - 5H_4u^2 - 5(2H_8 - H_4^2)u - 5(H_{12} - 25H_6^2)\}^2 - (128H_{20} + H_{10}^2)u = 0. \quad (4)$$

Hence, if

$$\begin{array}{lll} \alpha_1 = \tfrac{1}{2}[-(H_3 - \tau_3) + \surd\{(H_3 - \tau_3)^2 + 4(H_2 - \tau_2)(H_2^2 - \tau_2^2)\}] & & \\ \alpha_2 = & \alpha_3 = & \alpha_4 = \end{array}$$

then will

$$ax = -b + \omega\left\{\frac{\alpha_1^2\alpha_3}{(H_2 - \tau_2)^2}\right\}^{\frac{1}{5}} + \text{etc.}$$

THEOREM I.—Every algebraic equation A of degree n has a dioristic equation B of degree $(n - 2)!$ whose coefficients are seminvariants of the coefficients of A, and which is such that if A be solvable by radicals, B will have a rational root, and conversely, if B have a rational root, A will be solvable by radicals.

THEOREM II.—If the equation A be solvable by radicals, its coresolvent equations will be of the degrees whose indices are the prime factors of $n - 1$.

Examples.—The dioristic equation of the cubic

$$ax^3 + 3bx^2 + 3cx + d = 0$$

is

$$z + H_2 = 0,$$

and the coresolvent equation is a quadratic.

The dioristic equation of the quartic

$$ax^4 + 4bx^3 + 6cx^2 + 4dx + e = 0$$

is

$$4z^2 - H_3^2 = 0,$$

and the coresolvent equation is a cubic.

The dioristic equation of the quintic is reducible by a linear transformation to equation (4) above, and the coresolvent equations are two quadratics.

OTTAWA, CANADA, *March*, 1900.